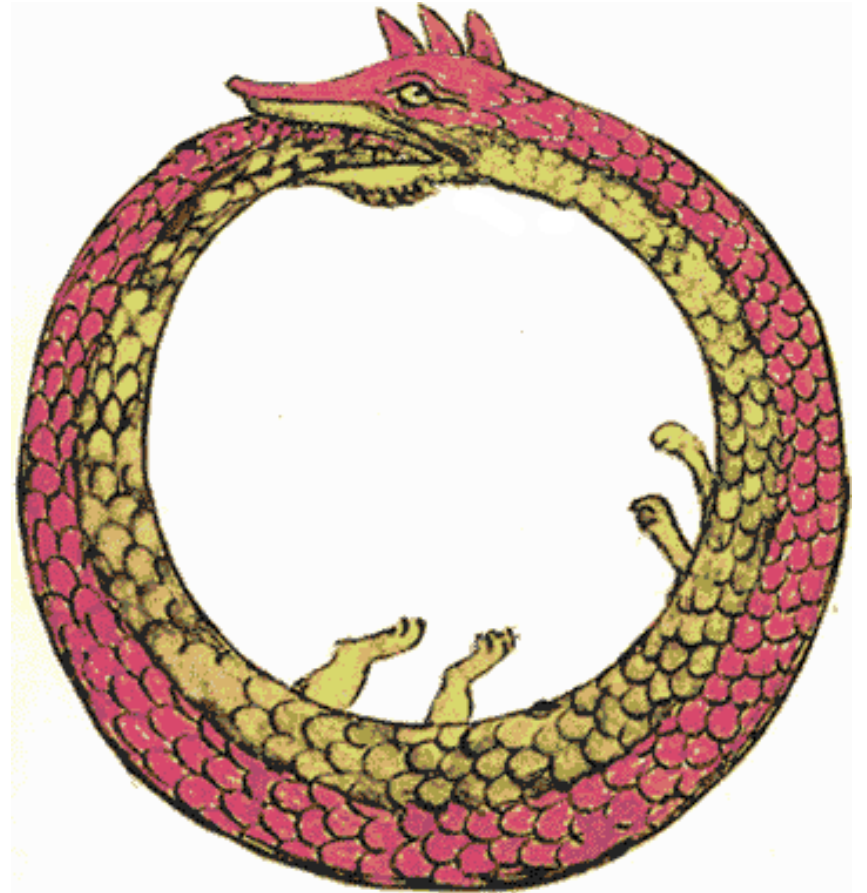


Tail Recursion



Problems with Recursion

- Recursion is generally favored over iteration in Scheme and many other languages
 - It's elegant, minimal, can be implemented with regular functions and easier to analyze formally
- It can also be less efficient
 - more functional calls and stack operations (context saving and restoration)
- Running out of stack space leads to failure
 - deep recursion

Tail recursion is iteration

- Tail recursion is a pattern of use that can be compiled or interpreted as iteration, avoiding the inefficiencies
- A tail recursive function is one where every recursive call is the last thing done by the function before returning and thus produces the function's value

Scheme's top level loop

- Consider a simplified version of the REPL

```
(define (repl)
```

```
  (printf "> ")
```

```
  (print (eval (read))))
```

```
(repl))
```

- This is an easy case: with no parameters there is not much context

Scheme's top level loop 2

- Consider a fancier REPL

```
(define (repl) (repl1 0))
```

```
(define (repl1 n)
```

```
  (printf "~s> " n)
```

```
  (print (eval (read))))
```

```
  (repl1 (add1 n)))
```

- This is only slightly harder: just modify the local variable `n` and start at the top

Scheme's top level loop 3

- There might be more than one tail recursive call

```
(define (repl1 n)
  (printf "~s> " n)
  (print (eval (read))))
(if (= n 9)
    (repl1 0)
    (repl1 (add1 n))))
```

- What's important is that there's nothing more to do in the function after the recursive calls

Two skills

- Distinguishing a trail recursive call from

Naïve recursive factorial

```
(define (fact1 n)
  ;; naive recursive factorial
  (if (< n 1)
      1
      (* n (fact1 (sub1 n)))))
```


Tail recursive factorial

```
(define (fact2 n)
  ; rewrite to just call the tail-recursive
  ; factorial with the appropriate initial values
  (fact2-helper n 1))

(define (fact2-helper n accumulator)
  ; tail recursive factorial calls itself as
  ; last thing to be done
  (if (< n 1)
      accumulator
      (fact2-helper (sub1 n) (* accumulator n))))
```

Trace shows what's going on

```
> (require (lib "trace.ss"))  
> (load "fact.ss")  
> (trace fact1)  
> (fact1 6)
```

```
| (fact1 6)  
| (fact1 5)  
| |(fact1 4)  
| |(fact1 3)  
| |(fact1 2)  
| |(fact1 1)  
| |(fact1 0)  
| | | 1  
| | | 1  
| | | 2  
| | 6  
| | 24  
| 120  
| 720  
720
```

fact2

```
> (trace fact2 fact2-helper)
> (fact2 6)
|(fact2 6)
|(fact2-helper 6 1)
|(fact2-helper 5 6)
|(fact2-helper 4 30)
|(fact2-helper 3 120)
|(fact2-helper 2 360)
|(fact2-helper 1 720)
|(fact2-helper 0 720)
|720
720
```

- Interpreter & compiler note the last expression to be evaled & returned in fact2-helper is a recursive call
- Instead of pushing state on the sack, it reassigns the local variables and jumps to beginning of the procedure
- Thus, the recursion is automatically transformed into iteration

Reverse a list

- This version works, but has two problems

```
(define (rev1 list)
```

```
  ; returns the reverse a list
```

```
  (if (null? list)
```

```
      empty
```

```
      (append (rev1 (rest list)) (list (first list))))))
```

- It is not tail recursive
- It creates needless temporary lists

A better reverse

```
(define (rev2 list) (rev2.1 list empty))
```

```
(define (rev2.1 list reversed)
```

```
  (if (null? list)
```

```
      reversed
```

```
      (rev2.1 (rest list)
```

```
              (cons (first list) reversed))))
```

```
> (load "reverse.ss")
> (trace rev1 rev2 rev2.1)
> (rev1 '(a b c))
|(rev1 (a b c))
| (rev1 (b c))
| |(rev1 (c))
| |(rev1 ())
| |()
| |(c)
| (c b)
|(c b a)
(c b a)
```

rev1 and rev2

```
> (rev2 '(a b c))
|(rev2 (a b c))
|(rev2.1 (a b c) ())
|(rev2.1 (b c) (a))
|(rev2.1 (c) (b a))
|(rev2.1 () (c b a))
|(c b a)
(c b a)
>
```

The other problem

- Append copies the top level list structure of its first argument.
- *(append '(1 2 3) '(4 5 6))* creates a copy of the list (1 2 3) and changes the last cdr pointer to point to the list (4 5 6)
- In reverse, each time we add a new element to the end of the list, we are (re-)copying the list.

Append (two args only)

```
(define (append list1 list2)
  (if (null? list1)
      list2
      (cons (first list1)
            (append (rest list1) list2))))
```


Why does this matter?

- The repeated rebuilding of the reversed list is needless work
- It uses up memory and adds to the cost of [garbage collection](#) (GC)
- GC adds a significant overhead to the cost of any system that uses it
- Experienced Lisp and Scheme programmers avoid algorithms that needlessly consume cons cells

Fibonacci

```
(define (fib n)
```

```
;; naive recursive fibonacci function
```

```
(if (< n 3) 1 (+ (fib (- n 1)) (fib (- n 2)))))
```

Run time for fib(n) $\cong O(2^n)$

Fibonacci

```
(define (fib2 n) (if (< n 3) 1 (fib-tr 3 n 1 1)))
```

```
(define (fib-tr n stop fib.n-2 fib.n-1 )
```

```
  (if (= n stop)
```

```
      (+ fib.n-1 fib.n-2)
```

```
      (fib-tr (+ 1 n) stop fib.n-1 (+ fib.n-1 fib.n-2))))
```

Run time for fib(n) \cong $O(n)$