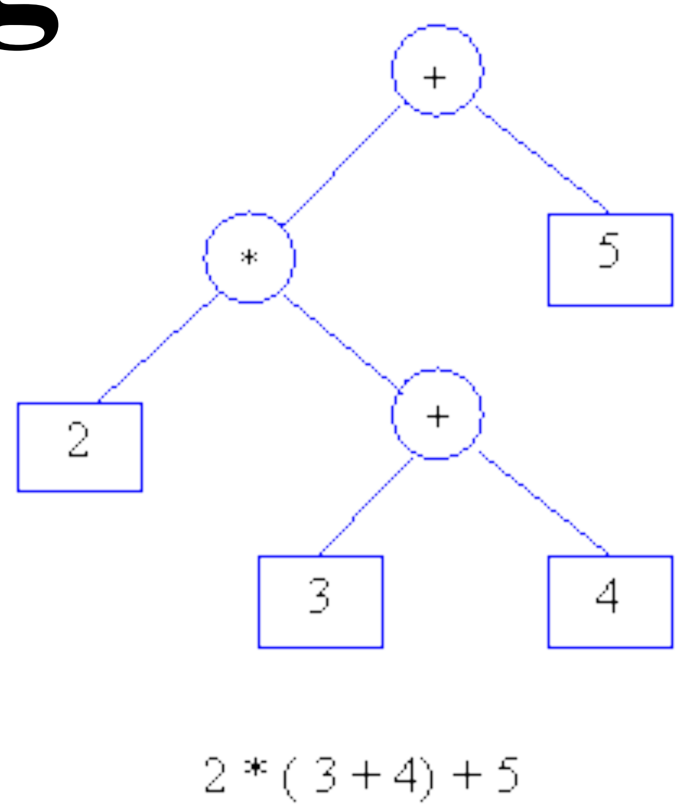
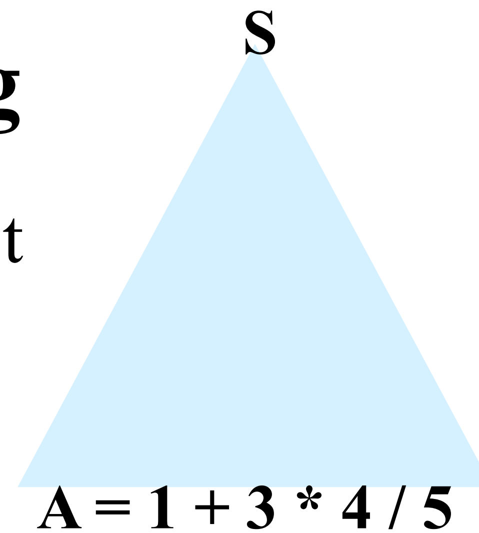
4 (c) parsing



Parsing

- A grammar describes syntactically legal strings in a language
- A *recogniser* simply accepts or rejects strings
- A *generator* produces strings
- A *parser* constructs a parse tree for a string
- Two common types of parsers:
  - bottom-up or data driven
  - top-down or hypothesis driven
- A *recursive descent parser* easily implements a top-down parser for simple grammars

Top down vs. bottom up parsing



- The parsing problem is to connect the root node S with the tree leaves, the input
- **Top-down parsers:** starts constructing the parse tree at the top (root) and move down towards the leaves. Easy to implement by hand, but requires restricted grammars. E.g.:
  - Predictive parsers (e.g., LL(k))
- **Bottom-up parsers:** build nodes on the bottom of the parse tree first. Suitable for automatic parser generation, handles larger class of grammars. E.g.:
  - shift-reduce parser (or LR(*k*) parsers)
- Both are general techniques that can be made to work for all languages (but not all grammars!).

Top down vs. bottom up parsing

- Both are general techniques that can be made to work for all languages (but not all grammars!)
- Recall that a given language can be described by several grammars
- Both of these grammars describe the same language

```
E -> E + Num
E -> Num
```

```
E -> Num + E
E -> Num
```

- The first one, with it's left recursion, causes problems for top down parsers
- For a given parsing technique, we may have to transform the grammar to work with it

Parsing complexity

- How hard is the parsing task? How to we measure that?
- Parsing an arbitrary CFG is $O(n^3)$ -- it can take time proportional the cube of the number of input symbols
  - This is bad! (why?)
- If we constrain the grammar somewhat, we can always parse in <u>linear</u> time. This is good! (why?)
- Linear-time parsing
  - LL parsers
    - Recognize LL grammar
    - Use a top-down strategy
  - LR parsers
    - Recognize LR grammar
    - Use a bottom-up strategy

- **LL(n) : Left to right, Leftmost derivation, look ahead at most n symbols.**
- **LR(n) : Left to right, Right derivation, look ahead at most n symbols.**

Top Down Parsing Methods

- Simplest method is a full-backup, *recursive descent* parser
- Often used for parsing simple languages
- Write recursive recognizers (subroutines) for each grammar rule
  - If rules succeeds perform some action (i.e., build a tree node, emit code, etc.)
  - If rule fails, return failure. Caller may try another choice or fail
  - On failure it "backs up"

Top Down Parsing Methods: Problems

- When going forward, the parser consumes tokens from the input, so what happens if we have to back up?
  - suggestions?
- Algorithms that use backup tend to be, in general, inefficient
  - There might be a large number of possibilities to try before finding the right one or giving up
- Grammar rules which are left-recursive lead to non-termination!

Recursive Decent Parsing: Example

For the grammar:

```
<term> -> <factor> {(*|/)<factor>}*
```

We could use the following recursive descent parsing subprogram (this one is written in C)

```
void term() {
  factor();     /* parse first factor*/
  while (next_token == ast_code ||
        next_token == slash_code) {
    lexical();  /* get next token */
    factor();   /* parse next factor */
  }
}
```

Problems

- Some grammars cause problems for top down parsers
- Top down parsers do not work with left-recursive grammars
  - E.g., one with a rule like: E -> E + T
  - We can transform a left-recursive grammar into one which is not
- A top down grammar can limit backtracking if it only has one rule per non-terminal
  - The technique of rule factoring can be used to eliminate multiple rules for a non-terminal

Left-recursive grammars

- A grammar is left recursive if it has rules like

```
X -> X β
```

- Or if it has indirect left recursion, as in

```
X -> A β
A -> X
```

- Q: Why is this a problem?
  - A: it can lead to non-terminating recursion!

Left-recursive grammars

- Consider

```
E -> E + Num
E -> Num
```

- We can manually or automatically rewrite a grammar removing left-recursion, making it ok for a top-down parser.

Elimination of Left Recursion

- Consider left-recursive grammar

  $S \rightarrow S\ \alpha$

  $S \rightarrow \beta$

- S generates strings

  β
  $\beta\ \alpha$
  $\beta\ \alpha\ \alpha$ …

- Rewrite using right-recursion

  $S \rightarrow \beta\ S'$

  $S' \rightarrow \alpha\ S' \mid \varepsilon$

- Concretely

  `T -> T + id`

  `T-> id`

- T generates strings

  id
  id+id
  id+id+id …

- Rewrite using right-recursion

  `T -> id`

  `T -> id T`

More Elimination of Left-Recursion

- In general

  $S \rightarrow S\ \alpha_1 \mid \ldots \mid S\ \alpha_n \mid \beta_1 \mid \ldots \mid \beta_m$

- All strings derived from S start with one of $\beta_1,\ldots,\beta_m$ and continue with several instances of $\alpha_1,\ldots,\alpha_n$
- Rewrite as

  $S \rightarrow \beta_1\ S' \mid \ldots \mid \beta_m\ S'$

  $S' \rightarrow \alpha_1\ S' \mid \ldots \mid \alpha_n\ S' \mid \varepsilon$

General Left Recursion

- The grammar

  $S \rightarrow A\ \alpha \mid \delta$

  $A \rightarrow S\ \beta$

  is also left-recursive because

  $S \rightarrow^{+} S\ \beta\ \alpha$

  where $\rightarrow^{+}$ means "can be rewritten in one or more steps"
- This indirect left-recursion can also be automatically eliminated

Summary of Recursive Descent

- Simple and general parsing strategy
  - Left-recursion must be eliminated first
  - … but that can be done automatically
- Unpopular because of backtracking
  - Thought to be too inefficient
- In practice, backtracking is eliminated by further restricting the grammar to allow us to successfully *predict* which rule to use

Predictive Parsers

- That there can be many rules for a non-terminal makes parsing hard
- A *predictive parser* processes the input stream typically from left to right
  - Is there any other way to do it? Yes for programming languages!
- It uses information from peeking ahead at the *upcoming terminal symbols* to decide which grammar rule to use next
- And *always* makes the right choice of which rule to use
- How much it can peek ahead is an issue

Predictive Parsers

- An important class of predictive parser only peek ahead one token into the stream
- An an *LL(k)* parser, does a **L**eft-to-right parse, a **L**eftmost-derivation, and **k-**symbol lookahead
- Grammars where one can decide which rule to use by examining only the *next* token are **LL(1)**
- LL(1) grammars are widely used in practice
  - The syntax of a PL can usually be adjusted to enable it to be described with an LL(1) grammar

Predictive Parser

Example: consider the grammar

$S \rightarrow$ **if** E **then** S **else** S
$S \rightarrow$ **begin** $S\ L$
$S \rightarrow$ **print** E
$L \rightarrow$ **end**
$L \rightarrow$ **;** $S\ L$
$E \rightarrow$ num = num

An *S* expression starts either with an IF, BEGIN, or PRINT token, and an *L* expression start with an END or a SEMICOLON token, and an *E* expression has only one production.

Remember…

- Given a grammar and a string in the language defined by the grammar …
- There may be more than one way to *derive* the string leading to the *same parse tree*
  - It depends on the order in which you apply the rules
  - And what parts of the string you choose to rewrite next
- All of the derivations are *valid*
- To simplify the problem and the algorithms, we often focus on one of two simple derivation strategies
  - A *leftmost* derivation
  - A *rightmost* derivation

LL(k) and LR(k) parsers

- Two important parser classes are LL(k) and LR(k)
- The name LL(k) means:
  - L: *Left-to-right* scanning of the input
  - L: Constructing *leftmost derivation*
  - k: max # of input symbols needed to predict parser action
- The name LR(k) means:
  - L: *Left-to-right* scanning of the input
  - R: Constructing *rightmost derivation* in reverse
  - k: max # of input symbols needed to select parser action
- A LR(1) or LL(1) parser never need to *"look ahead"* more than *one* input token to know what parser production rule applies

Predictive Parsing and Left Factoring

- Consider the grammar

```
E → T + E
E → T
T → int
T → int * T
T → ( E )
```

Even left recursion is removed, a grammar may not be parsable with a LL(1) parser

- Hard to predict because
  - For T, two productions start with *int*
  - For E, it is not clear how to predict which rule to use
- Must **left-factored** grammar before use for predictive parsing
- Left-factoring involves rewriting rules so that, if a non-terminal has > 1 rule, each begins with a **terminal**

Left-Factoring Example

Add new non-terminals X and Y to factor out **common prefixes** of rules

```
E → T + E
E → T
T → int
T → int * T
T → ( E )
```

```
E → T X
X → + E
X → ε
T → ( E )
T → int Y
Y → * T
Y → ε
```

For each non-terminal the revised grammar, there is either only one rule or every rule begins with a terminal or ε

Left Factoring

- Consider a rule of the form

  A => a B1 | a B2 | a B3 | … a Bn
- A top down parser generated from this grammar is not efficient as it requires backtracking.
- To avoid this problem we left factor the grammar.
  - Collect all productions with the same left hand side and begin with the same symbols on the right hand side
  - Combine common strings into a single production and append a new non-terminal to end of this new production
  - Create new productions using this new non-terminal for each of the suffixes to the common production.
- After left factoring the above grammar is transformed into:

  A –> a A1

  A1 -> B1 | B2 | B3 … Bn

Using Parsing Tables

- LL(1) means that for each non-terminal and token there is only **one** production
- Can be represented as a simple table
  – One dimension for current non-terminal to expand
  – One dimension for next token
  – A table entry contains one rule's action or empty if error
- Method similar to recursive descent, except
  – For each non-terminal S
  – We look at the next token *a*
  – And chose the production shown at table cell [S, a]
- Use a stack to keep track of pending non-terminals
- Reject when we encounter an error state, accept when we encounter end-of-input

LL(1) Parsing Table Example

Left-factored grammar

```
E → T X
X → + E | ε
T → ( E ) | int Y
Y → * T | ε
```

End of input symbol

The LL(1) parsing table

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

LL(1) Parsing Table Example

```
E → T X
X → + E | ε
T → ( E ) | int Y
Y → * T | ε
```

- Consider the **[E, int]** entry
  - "When current non-terminal is E & next input *int*, use production E→ T X"
  - It's the only production that can generate an *int* in next place
- Consider the **[Y, +]** entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - Y can be followed by + only in a derivation where Y→ε
- Consider the **[E, *]** entry
  - Blank entries indicate error situations
  - "There is no way to derive a string starting with * from non-terminal E"

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

LL(1) Parsing Algorithm

```
initialize stack = <S $> and next
repeat
  case stack of
    <X, rest>  : if T[X,*next] = Y1...Yn
                     then stack ← <Y1... Yn rest>;
                     else  error ();
    <t, rest>  : if t == *next ++
                     then  stack ← <rest>;
                     else error ();
until stack == < >
```

where:

(1) next points to the next input token
(2) X matches some non-terminal
(3) t matches some terminal

LL(1) Parsing Example

Stack	Input	Action
E $	int * int $	pop();push(T X)
T X $	int * int $	pop();push(int Y)
int Y X $	int * int $	pop();next++
Y X $	* int $	pop();push(* T)
* T X $	* int $	pop();next++
T X $	int $	pop();push(int Y)
int Y X $	int $	pop();next++;
Y X $	$	pop()
X $	$	pop()
$	$	ACCEPT!

E → TX
X → +E
X → ε
T → (E)
T → int Y
Y → *T
Y → ε

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

Constructing Parsing Tables

- No table entry can be multiply defined
- If $A \rightarrow \alpha$, where in the line of A we place α ?
- In column t where t can start a string derived from α
  - $\alpha \rightarrow^* t\ \beta$
  - We say that $t \in \text{First}(\alpha)$
- In the column t if α is ε and t can follow an A
  - $S \rightarrow^* \beta\ A\ t\ \delta$
  - We say $t \in \text{Follow}(A)$

Computing First Sets

Definition: $\text{First}(X) = \{t \mid X \to^* t\alpha\} \cup \{\varepsilon \mid X \to^* \varepsilon\}$

Algorithm sketch (see book for details):

1. for all terminals t do $\text{First}(t) \leftarrow \{ t \}$
2. for each production $X \to \varepsilon$ do $\text{First}(X) \leftarrow \{ \varepsilon \}$
3. if $X \to A_1 \ldots A_n\ \alpha$ and $\varepsilon \in \text{First}(A_i)$, $1 \le i \le n$ do add First(α) to First(X)
4. for each $X \to A_1 \ldots A_n$ s.t. $\varepsilon \in \text{First}(A_i)$, $1 \le i \le n$ do add ε to First(X)
5. repeat steps 4 and 5 until no First set can be grown

First Sets. Example

Recall the grammar

$E \to T X$ $\qquad X \to + E \mid \varepsilon$

$T \to (E) \mid \text{int } Y$ $\qquad Y \to * T \mid \varepsilon$

First sets

First(( ) = { (} $\qquad$ First(T) = {int, (}

First( )) = {) } $\qquad$ First(E) = {int, (}

First(int) = { int } $\qquad$ First(X) = {+, ε }

First(+) = { + } $\qquad$ First(Y) = {*, ε }

First(*) = { * }

Computing Follow Sets

- Definition:

  $$\text{Follow}(X) = \{\ t \mid S \rightarrow^{*} \beta\ X\ t\ \delta\ \}$$

- Intuition
  - If S is the start symbol then $\$ \in \text{Follow}(S)$
  - If $X \rightarrow A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$ and
    $\text{Follow}(X) \subseteq \text{Follow}(B)$
  - Also if $B \rightarrow^{*} \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$

Computing Follow Sets

Algorithm sketch:

1. $\text{Follow}(S) \leftarrow \{ \$ \}$
2. For each production $A \rightarrow \alpha\ X\ \beta$
   - add $\text{First}(\beta) - \{\varepsilon\}$ to $\text{Follow}(X)$
3. For each $A \rightarrow \alpha\ X\ \beta$ where $\varepsilon \in \text{First}(\beta)$
   - add $\text{Follow}(A)$ to $\text{Follow}(X)$

- repeat step(s) ___ until no Follow set grows

Follow Sets. Example

- Recall the grammar

$E \to T\ X \qquad\qquad X \to +\ E \mid \varepsilon$

$T \to (\ E\ ) \mid \text{int}\ Y \qquad\qquad Y \to *\ T \mid \varepsilon$

- Follow sets

Follow(+) = { int, (}   Follow(*) = { int, (}

Follow(( ) = { int, (}   Follow(E) = {), $}

Follow(X) = {$,) }   Follow(T) = {+,) , $}

Follow( )) = {+,) , $}   Follow(Y) = {+,) , $}

Follow(int) = {*, +,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G
- For each production $A \rightarrow \alpha$ in G do:
  - For each terminal $t \in \text{First}(\alpha)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in \text{First}(\alpha)$, for each $t \in \text{Follow}(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in \text{First}(\alpha)$ and $\$ \in \text{Follow}(A)$ do
    - $T[A, \$] = \alpha$

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
- Reasons why a grammar is not LL(1) include
  - G is ambiguous
  - G is left recursive
  - G is not left-factored
- Most programming language grammars are not strictly LL(1)
- There are tools that build LL(1) tables

Bottom-up Parsing

- YACC uses bottom up parsing. There are two important operations that bottom-up parsers use: **shift** and **reduce**
  - In abstract terms, we do a simulation of a Push Down Automata as a finite state automata
- Input: given string to be parsed and the set of productions.
- Goal: Trace a rightmost derivation in reverse by starting with the input string and working backwards to the start symbol

Algorithm

1. Start with an empty stack and a full input buffer. (The string to be parsed is in the input buffer.)
2. Repeat until the input buffer is empty and the stack contains the start symbol.

   a. <u>Shift</u> zero or more input symbols onto the stack from input buffer until a handle (beta) is found on top of the stack. If no handle is found report syntax error and exit.

   b. <u>Reduce</u> handle to the nonterminal A. (There is a production *A* -> *beta*)
3. <u>Accept</u> input string and return some representation of the derivation sequence found (e.g.., <u>parse tree)</u>

- The four key operations in bottom-up parsing are <u>shift, reduce, accept</u> and <u>error.</u>
- Bottom-up parsing is also referred to as shift-reduce parsing.
- Important thing to note is to know when to shift and when to reduce and to which reduce.

Example of Bottom-up Parsing

STACK	INPUT BUFFER	ACTION
$	num1+num2*num3$	shift
$num1	+num2*num3$	reduc
$F	+num2*num3$	reduc
$T	+num2*num3$	reduc
$E	+num2*num3$	shift
$E+	num2*num3$	shift
$E+num2	*num3$	reduc
$E+F	*num3$	reduc
$E+T	*num3$	shift
E+T*	num3$	shift
E+T*num3	$	reduc
E+T*F	$	reduc
E+T	$	reduc
E	$	accept

E -> E+T
| T
| E-T
T -> T*F
| F
| T/F
F -> (E)
| id
| -E
num