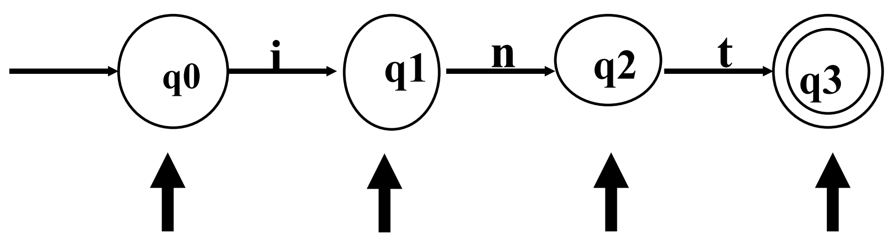
4b

Lexical analysis
Finite Automata

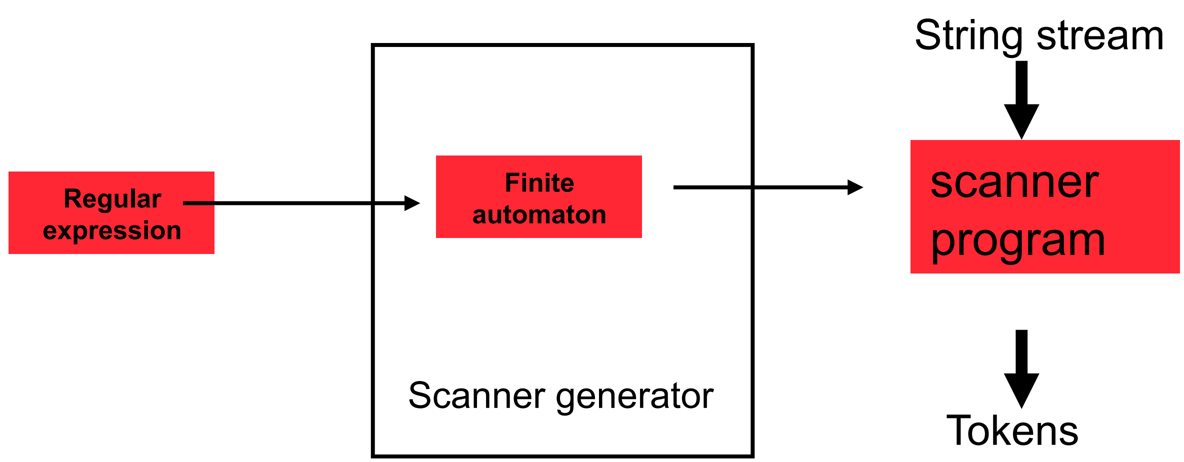
Finite Automata (FA)

- FA also called Finite State Machine (FSM)
  - Abstract model of a computing entity.
  - Decides whether to accept or reject a string.
  - Every regular expression can be represented as a FA and vice versa
- Two types of FAs:
  - Non-deterministic (NFA): Has more than one alternative action for the same input symbol.
  - Deterministic (DFA): Has at most one action for a given input symbol.
- Example: how do we write a program to recognize the Java keyword “int”?



RE and Finite State Automaton (FA)

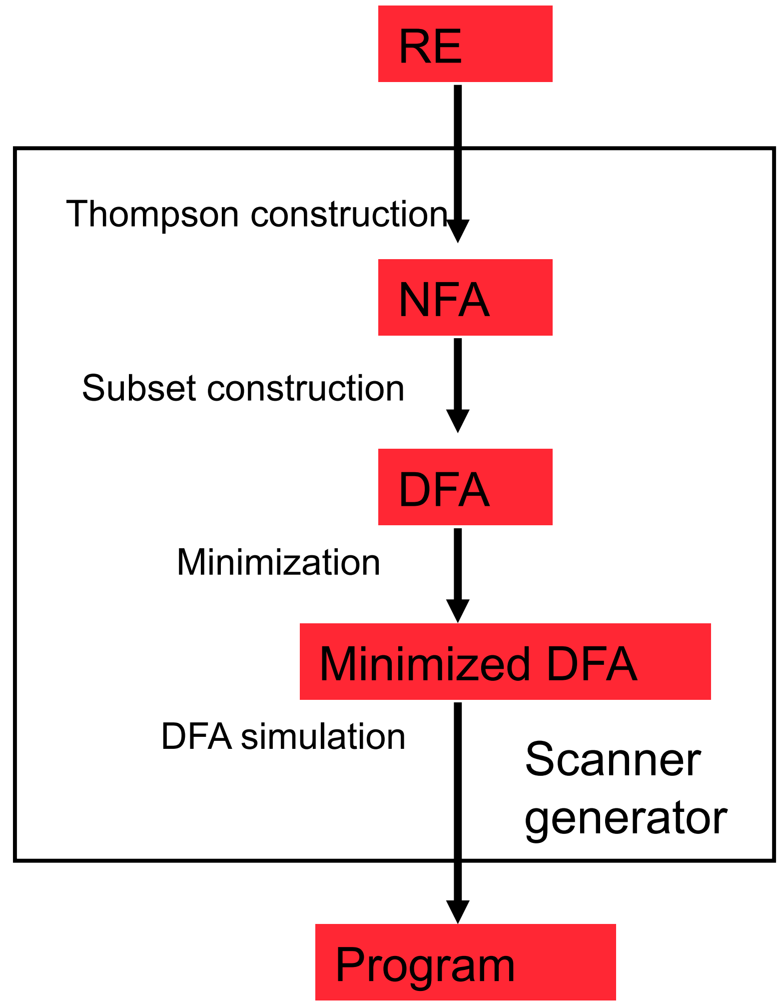
- Regular expressions are a declarative way to describe the tokens
  - Describes *what* is a token, but not *how* to recognize the token
- FAs are used to describe *how* the token is recognized
  - FAs are easy to simulate in a programs
- There is a 1-1 correspondence between FAs & regular expressions
  - A scanner generator (e.g., lex) bridges the gap between regular expressions and FAs.



Inside scanner generator

Main components of scanner generation (e.g., Lex)

- Convert a regular expression to a non-deterministic finite automaton (NFA)
- Convert the NFA to a determinstic finite automaton (DFA)
- Improve the DFA to minimize the number of states
- Generate a program in C or some other language to "simulate" the DFA



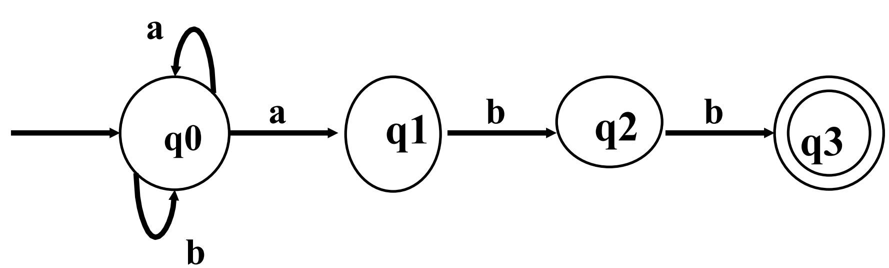
Non-deterministic Finite Automata (FA)

- NFA (Non-deterministic Finite Automaton) is a 5-tuple (S, Σ, δ, $S0$, F):
  - S: a set of states;
  - Σ: the symbols of the input alphabet;
  - δ : a set of transition functions;
    - » move(state, symbol) → a set of states
  - S0: $s0 \in S$, the start state;
  - F: $F \subseteq S$, a set of final or accepting states.
- Non-deterministic -- a state and symbol pair can be mapped to a set of states.
- Finite—the number of states is finite.

Transition Diagram

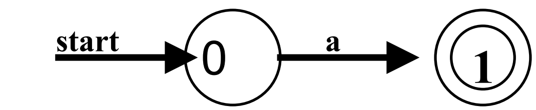
- FA can be represented using transition diagram.
- Corresponding to FA definition, a transition diagram has:
  - **States** represented by circles;
  - An **Alphabet** (Σ) represented by labels on edges;
  - **Transitions** represented by labeled directed edges between states. The label is the input symbol;
  - One **Start State** shown as having an arrow head;
  - One or more **Final State**(s) represented by double circles.

- Example transition diagram to recognize (a|b)*abb

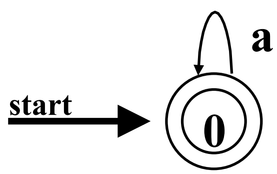


Simple examples of FA

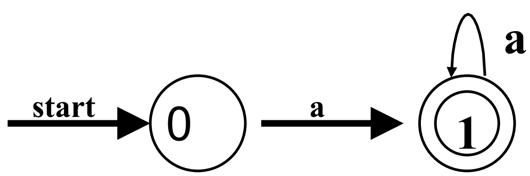
a



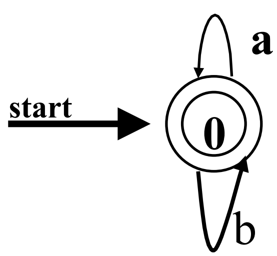
a*

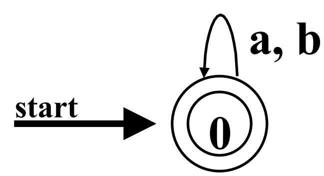


a+



(a|b)*



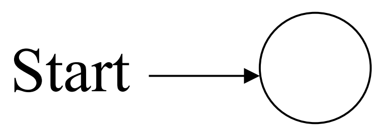


Procedures of defining a DFA/NFA

- Defining input alphabet and initial state
- Draw the transition diagram
- Check
  - Do all states have out-going arcs labeled with all the input symbols (DFA)
  - Any missing final states?
  - Any duplicate states?
  - Can all strings in the language can be accepted?
  - Are any strings not in the language accepted?
- Naming all the states
- Defining $(S, \Sigma, \delta, q_0, F)$

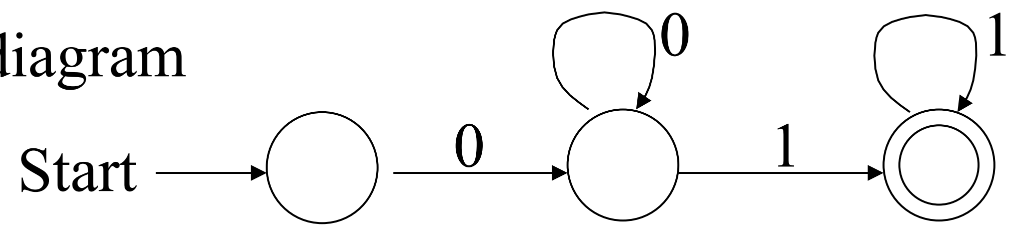
Example of constructing a FA

- Construct a DFA that accepts a language L over the alphabet {0, 1} such that L is the set of all strings with *any* number of "0"s followed by *any* number of "1"s.
- Regular expression: 0*1*
- $\Sigma = \{0, 1\}$
- Draw initial state of the transition diagram

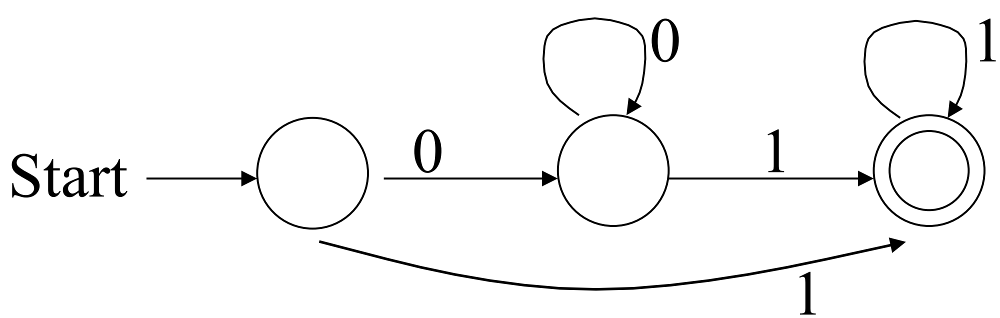


Example of constructing a FA

- Draft the transition diagram

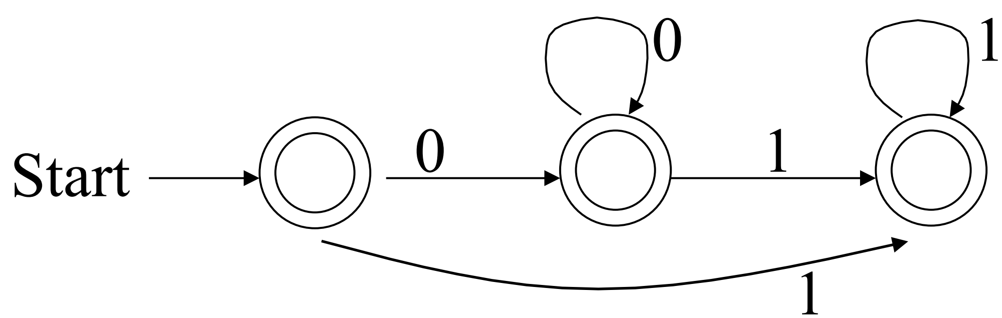


- Is “111” accepted?
- The leftmost state has missed an arc with input “1”



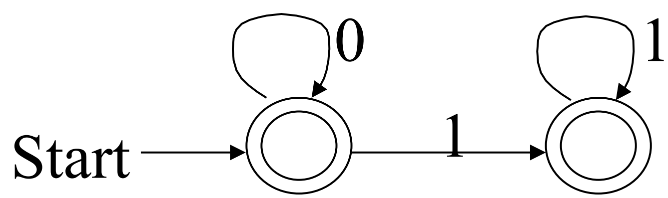
Example of constructing a FA

- Is “00” accepted?
- The leftmost two states are also final states
  - First state from the left: ε is also accepted
  - Second state from the left:
    strings with “0”s only are also accepted

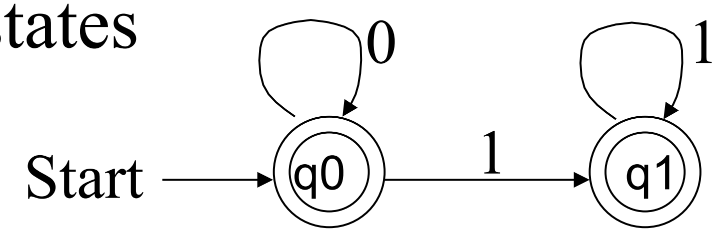


Example of constructing a FA

- The leftmost two states are duplicate
  - their arcs point to the same states with the same symbols

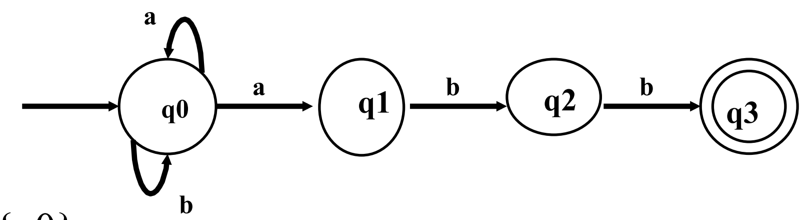


- Check that they are correct
  - All strings in the language can be accepted
    - » ε, the empty string, is accepted
    - » strings with “0”s / “1”s only are accepted
  - No strings not in language are accepted
- Naming all the states



How does a FA work

- NFA definition for **(a|b)*abb**
  - S = {q0, q1, q2, q3 }
  - Σ = { a, b }
  - Transitions: move(q0,a)={q0, q1}, move(q0,b)={q0}, ...
  - s0 = q0
  - F = { q3 }



- Transition diagram representation
  - Non-determinism:
    - » exiting from one state there are multiple edges labeled with same symbol, or
    - » There are epsilon edges.
  - How does FA work? Input: ababb

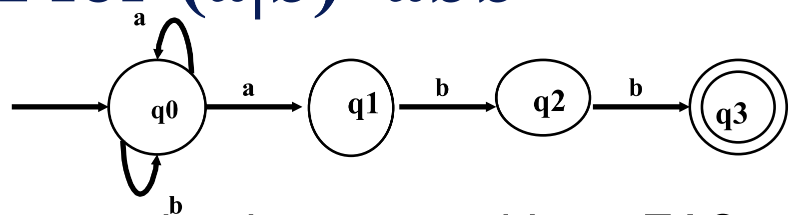
move(0, a) = 1
move(1, b) = 2
move(2, a) = ? (undefined)

REJECT !

move(0, a) = 0
move(0, b) = 0
move(0, a) = 1
move(1, b) = 2
move(2, b) = 3

ACCEPT !

FA for (a|b)*abb



– What does it mean that a string is accepted by a FA?

An FA accepts an input string *x* iff there is a path from start to a final state, such that the edge labels along this path spell out *x*;

– A path for “aabb”: $Q0 \rightarrow^{a} q0 \rightarrow^{a} q1 \rightarrow^{b} q2 \rightarrow^{b} q3$

– Is “aab” acceptable?

$Q0 \rightarrow^{a} q0 \rightarrow^{a} q1 \rightarrow^{b} q2$

$Q0 \rightarrow^{a} q0 \rightarrow^{a} q0 \rightarrow^{b} q0$

»Final state must be reached;

»In general, there could be several paths.

– Is “aabbb” acceptable?

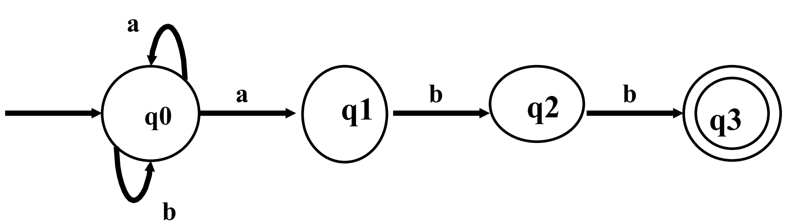
$Q0 \rightarrow^{a} q0 \rightarrow^{a} q1 \rightarrow^{b} q2 \rightarrow^{b} q3$

»Labels on the path must spell out the entire string.

Transition table

- A transition table is a good way to implement a FSA
  - One row for each state, S
  - One column for each symbol, A
  - Entry in cell (S,A) gives set of states can be reached from state S on input A
- A Nondeterministic Finite Automaton (NFA) has at least one cell with more than one state
- A Deterministic Finite Automaton (DFA) has a singe state in every cell

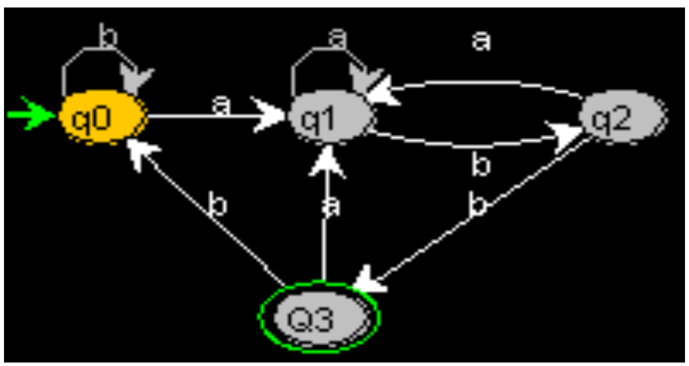
(a|b)*abb



STATES	INPUT	
	a	b
>Q0	{q0, q1}	q0
Q1		q2
Q2		q3
*Q3		

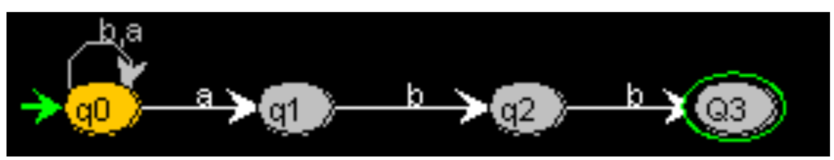
DFA (Deterministic Finite Automaton)

- A special case of NFA where the transition function maps the pair (state, symbol) to one state.
  - When represented by transition diagram, for each state *S* and symbol *a,* there is at most one edge labeled *a* leaving *S;*
  - When represented transition table, each entry in the table is a single state.
  - There are no ε-transition
- Example: DFA for (a|b)*abb



STATES	INPUT	
	a	b
q0	q1	q0
q1	q1	q2
q2	q1	q3
q3	q1	q0

- **Recall the NFA:**



DFA to program

- NFA is more concise, but not as easy to implement;
- In DFA, since transition tables don’t have any alternative options, DFAs are easily simulated via an algorithm.
- Every NFA can be converted to an equivalent DFA
  - What does equivalent mean?
- There are general algorithms that can take a DFA and produce a “minimal” DFA.
  - Minimal in what sense?
- There are programs that take a regular expression and produce a program based on a minimal DFA to recognize strings defined by the RE.
- You can find out more in 451 (automata theory) and/or 431 (Compiler design)