## $4 a$

## Lexical analysis

## Concepts

- Overview of syntax and semantics
- Step one: lexical analysis
-Lexical scanning
-Regular expressions
-DFAs and FSAs
-Lex



## Lexical analysis in perspective

LEXICAL ANALYZER: Transforms character stream to token stream

- Also called scanner, lexer, linear analysis



## LEXICAL ANALYZER

- Scans Input
- Removes whitespace, newlines, ...
- Identifies Tokens
- Creates Symbol Table
- Inserts Tokens into symbol table
- Generates Errors
- Sends Tokens to Parser


## PARSER

- Performs Syntax Analysis
- Actions Dictated by Token Order
- Updates Symbol Table Entries
- Creates Abstract Rep. of Source
- Generates Errors


## Where we are



## Basic lexical analysis terms

- Token
- A classification for a common set of strings
- Examples: <identifier>, <number>, <operator>, <open paren>, etc.
- Pattern
- The rules which characterize the set of strings for a token
- Recall file and OS wildcards (*.java)
- Lexeme
- Actual sequence of characters that matches pattern and is classified by a token
- Identifiers: x , count, name, etc...
- Integers: -12, 101, $0, \ldots$


## Examples of token, lexeme and pattern

## if (price + gst - rebate <= 10.00) gift := false

| Token | lexeme | Informal description of pattern |
| :--- | :--- | :--- |
| if | if | if |
| Lparen | $($ | $($ |
| Identifier | price | String consists of letters and numbers and starts with a letter |
| operator | + | + |
| identifier | gst | String consists of letters and numbers and starts with a letter |
| operator | - | - |
| identifier | rebate | String consists of letters and numbers and starts with a letter |
| Operator | $<=$ | Less than or equal to |
| constant | $\mathbf{1 0 . 0 0}$ | Any numeric constant |
| rparen | $)$ | ) |
| identifier | gift | String consists of letters and numbers and starts with a letter |
| Operator | $:=$ | Assignment symbol |
| identifier | false | String consists of letters and numbers and starts with a letter |

## Regular expression (REs)

- Scanners are based on regular expressions that define simple patterns
- Simpler and less expressive than BNF
- Examples of a regular expression letter: a|b|c|...|z|A|B|C...|Z digit: $0|1| 2|3| 4|5| 6|7| 8 \mid 9$ identifier: letter (letter | digit)*
- Basic operations are (1) set union, (2) concatenation and (3) Kleene closure
- Plus: parentheses, naming patterns
- No recursion!


## Regular expression (REs)

## Example

letter: a|b|c|...|z|A|B|C...|Z
digit: $0|1| 2|3| 4|5| 6|7| 8 \mid 9$
identifier: letter (letter | digit)*

## letter ( letter | digit ) *

letter ( letter | digit) )
letter ( letter| digit)*
concatenation: one pattern followed by another
set union: one pattern or another

Kleene closure: zero or more repetions of a pattern


Regular expressions are extremely useful in many applications. Mastering them will serve you well.

## Formal language operations

| Operation | Notation | Definition | Example $L=\{a, b\} \quad M=\{0,1\}$ |
| :---: | :---: | :---: | :---: |
| union of L and M | $L \cup M$ | $\mathrm{L} \cup \mathrm{M}=\{\mathrm{s} \mid \mathrm{s}$ is in L or s is in $M$ \} | $\{a, b, 0,1\}$ |
| concatenation of L and M | LM | $\begin{gathered} L M=\{s t \mid s \text { is in } L \text { and } t \text { is } \\ \text { in } M\} \end{gathered}$ | \{a0, a1, b0, b1\} |
| Kleene closure of $L$ | L* | L* denotes zero or more concatenations of L | All the strings consists of "a" and "b", plus the empty string. $\{\varepsilon, a, b, a a, b b, a b, b a$, aaa, ...\} |
| positive closure | L+ | L+ denotes "one or more concatenations of " $L$ | All the strings consists of "a" and "b". \{a, b, aa, bb, ab, ba, aaa, ...\} |

## Regular expression

- Let $\Sigma$ be an alphabet, $r$ a regular expression then $\mathrm{L}(r)$ is the language that is characterized by the rules of $r$
- Definition of regular expression
$-\varepsilon$ is a regular expression that denotes the language $\{\varepsilon\}$
- If a is in $\Sigma$, a is a regular expression that denotes $\{a\}$
- Let $r$ \& $s$ be regular expressions with languages $L(r) \& L(s)$
» $(\mathrm{r}) \mid(\mathrm{s})$ is a regular expression $\rightarrow \mathrm{L}(\mathrm{r}) \cup \mathrm{L}(\mathrm{s})$
" $(\mathrm{r})(\mathrm{s})$ is a regular expression $\rightarrow \mathrm{L}(\mathrm{r}) \mathrm{L}(\mathrm{s})$
$»(\mathrm{r})^{*}$ is a regular expression $\rightarrow(\mathrm{L}(\mathrm{r}))^{*}$
- It is an inductive definition!
- A regular language is a language that can be defined by a regular expression


## Regular expression example revisited

- Examples of regular expression

Letter: a|b|c|...|z|A|B|C...|Z
Digit: 0|1|2|3|4|5|6|7|8|9
Identifier: letter (letter | digit)*

- Q: why it is an regular expression?
- Because it only uses the operations of union, concatenation and Kleene closure
- Being able to name patterns is just syntactic sugar
- Using parentheses to group things is just syntactic sugar provided we specify the precedence and associatively of the operators (i.e., |, * and "concat")


## Another common operator: +

- The + operator is commonly used to mean "one or more repetitions" of a pattern
- For example, letter ${ }^{+}$means one or more letters
- We can always do without this, e.g.
letter ${ }^{+}$is equivalent to letter letter*
- So the + operator is just syntactic sugar


## Precedence of operators

In interpreting a regular expression

- Parens scope sub-expressions
-     * and + have the highest precedence
- Concanenation comes next
- | is lowest.
- All the operators are left associative
- Example
$-(\mathrm{A}) \mid\left((\mathrm{B})^{*}(\mathrm{C})\right)$ is equivalent to $\mathrm{A} \mid \mathrm{B} * \mathrm{C}$
- What strings does this generate or match?

Either an $A$ or any number of Bs followed by a C

## Epsilon

- Sometimes we'd like a token that represents nothing
- This makes a regular expression matching more complex, but can be useful
- We use the lower case Greek letter epsilon, $\varepsilon$, for this special token
- Example:
digit: $0|1| 2|3| 4|5| 6|7| 8|9| 0$
sign: $+|-| \varepsilon$
int: sign digit+


## Properties of regular expressions

We can easily determine some basic properties of the operators involved in building regular expressions

| Property | Description |
| :--- | :--- |
| $\mathbf{r}\|\mathbf{s}=\mathbf{s}\| \mathbf{r}$ | $\mid$ is commutative |
| $r\|(\mathbf{s} \mid \mathbf{t})=(\mathbf{r} \mid \mathbf{s})\| \mathbf{t}$ | $\mid$ is associative |
| $(\mathrm{rs}) \mathbf{t}=\mathrm{r}(\mathbf{s t})$ | Concatenation is associative |
| $\mathrm{r}(\mathbf{s} \mid \mathbf{t})=\mathrm{rs} \mid \mathbf{r t}$ <br> $(\mathbf{s} \mid \mathbf{t}) \mathrm{r}=\mathbf{s r} \mid \mathbf{t r}$ | Concatenation distributes over \| |
| $\ldots \ldots$ | $\ldots$ |

## Notational shorthand of regular expression

- One or more instance
$-\mathrm{L}+=\mathrm{L} \mathrm{L}^{*}$
- L* $=\mathrm{L}+\mid \varepsilon$
- Examples
" digits: digit digit*
» digits: digit+


## More syntatic sugar

- Zero or one instance
-L ? $=\mathrm{L} \mid \varepsilon$
- Examples
» Optional_fraction $\rightarrow$.digits $\mid \varepsilon$
$»$ optional_fraction $\rightarrow$ (.digits)?
- Character classes
$-[a b c]=a|b| c$
$-[a-z]=a|b| c . . . \mid z$


## Regular grammar and regular expression

- They are equivalent
-Every regular expression can be expressed by regular grammar
-Every regular grammar can be expressed by regular expression
- Example
- An identifier must begin with a letter and can be followed by arbitrary number of letters and digits.

| Regular expression | Regular grammar |
| :---: | :---: |
| ID: LETTER (LETTER \| DIGIT)* | ID $\rightarrow$ LETTER ID_REST <br> ID_REST $\rightarrow$ LETTER ID_REST <br> \| DIGIT ID_REST <br> \| EMPTY |
|  |  |

## Formal definition of tokens

- A set of tokens is a set of strings over an alphabet $\{$ read, write, $+,-, *, /,:=, 1,2, \ldots, 10, \ldots, 3.45 \mathrm{e}-3, \ldots\}$
- A set of tokens is a regular set that can be defined by using a regular expression
- For every regular set, there is a finite automaton (FA) that can recognize it
- Aka deterministic Finite State Machine (FSM)
$-i . e$ determine whether a string belongs to the set or not
- Scanners extract tokens from source code in the same way DFAs determine membership


## $\mathbf{F S M}=\mathbf{F A}$

- Finite state machine and finite automaton are different names for the same concept
- The basic concept is important and useful in almost every aspect of computer science
- The concept provides an abstract way to describe a process that
- Has a finite set of states it can be in
- Gets a sequence of inputs
- Each input causes the process to go from its current state to a new state (which might be the same!)
- If after the input ends, we are in one of a set of accepting state, the input is accepted by the FA


## Example

This example shows a FA that determines whether a binary number has an odd or even number of 0's, where S1 is an accepting state.


## Deterministic finite automaton (DFA)

- In a DFA there is only one choice for a given input in every state
- There are no states with two arcs that match the same input that transition to different states



## Deterministic finite automaton (DFA)

- If there is an input symbol that matches no arc for the current state, the input is not accepted
- This FA accepts only binary numbers that are multiples of three
- SO is both the start state and an accept state.



## REs can be represented as DFAs

Regular expression for a simple identifier

```
Letter: a|b|c|...|z|A|B|C...|Z
Digit: 0|1|2|3|4|5|6|7|8|9
Identifier: letter (letter | digit)*
```

- Incoming arrow identifies a single start state
-     * marks a possible final (accepting) state

- State transitions enabled by input
- Arcs represent transitions and are labeled with required input


## REs can be represented as DFAs

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## Token Definition Example

Numeric literals in Pascal, e.g.
$1,123,3.1415,10 \mathrm{e}-3,3.14 \mathrm{e} 4$
Definition of token unsignedNum
$D I G \rightarrow 0|1| 2|3| 4|5| 6|7| 8 \mid 9$
unsignedInt $\rightarrow$ DIG DIG*
unsignedNum $\rightarrow$ unsignedInt
$(($. unsignedInt) $\mid \varepsilon)$
$((\mathrm{e}(+|-| \varepsilon)$ unsignedInt $) \mid \varepsilon)$

## Note:

- Recursion restricted to leftmost or rightmost position on LHS
- Parentheses used to avoid ambiguity
- It's always possible to rewrite by removing epsilons ( $\varepsilon$ )

- Accepting states marked with a *
-FAs with epsilons are nondeterministic
- NFAs are harder to implement, use backtracking
- Every NFA can be rewritten as a DFA (gets larger, tho)


## Simple Problem

- Write a C program which reads in a character string, consisting of a's and b's, one character at a time. If the string contains a double aa, then print string accepted else print string rejected.
- An abstract solution to this can be expressed as a DFA


The state transitions of a DFA can be encoded as a table which specifies the new state for a given current state and input


```
#include <stdio.h>
main()
{ enum State {S1, S2, S3};
    enum State currentState = S1;
    int c = getchar();
    while (c != EOF) {
        switch(currentState) {
        case S1: if (c == 'a') currentState = S2;
        if (c == 'b') currentState = S1;
                break;
    case S2: if (c == 'a') currentState = S3;
                        if (c == 'b') currentState = S1;
                        break;
    case S3: break;
        }
    c = getchar();
    }
    if (currentState == S3) printf("string accepted\n");
    else printf("string rejected\n");
}
```

```
#include <stdio.h>
main()
{ enum State {S1, S2, S3};
    enum Label {A, B};
program
    enum State currentState = S1;
    enum State table[3][2] = {{S2, S1}, {S3, S1}, {S3, S3}};
    int label;
    int c = getchar();
    while (c != EOF) {
        if (c == 'a') label = A;
        if (c == 'b') label = B;
        currentState = table[currentState][label];
        c = getchar();
    }
    if (currentState == S3) printf("string accepted\n");
    else printf("string rejected\n");
}
```

using a table
simplifies the

## Lex

- Lexical analyzer generator
- It writes a lexical analyzer
- Assumption
- each token matches a regular expression
- Needs
- set of regular expressions
- for each expression an action
- Produces
- A C program
- Automatically handles many tricky problems
- flex is the gnu version of the venerable unix tool lex.
- Produces highly optimized code


## Scanner Generators

- E.g. lex, flex
- These programs take a table as their input and return a program (i.e. a scanner) that can extract tokens from a stream of characters
- A very useful programming utility, especially when coupled with a parser generator (e.g., yacc)
- standard in Unix



## Lex example



## Examples

## - The examples to follow can be access on gl <br> - See /afs/umbc.edu/users/f/i/finin/pub/lex

```
% ls -l /afs/umbc.edu/users/f/i/finin/pub/lex
total 8
drwxr-xr-x 2 finin faculty 2048 Sep 27 13:31 aa
drwxr-xr-x 2 finin faculty 2048 Sep 27 13:32 defs
drwxr-xr-x 2 finin faculty 2048 Sep 27 11:35 footranscanner
drwxr-xr-x 2 finin faculty 2048 Sep 27 11:34 simplescanner
```


## A Lex Program

## ... definitions

DIG [0-9]
ID [a-z][a-z0-9]*
\% \%
$\{D I G\}+\quad$ printf("Integer\n");
$\{D I G\}+" . "\{D I G\} * \operatorname{printf}($ "Float $\backslash n ") ;$
\{ID \}
[ $\backslash$ t $\backslash n]+$
.
$\% \%$
main() $\{$ yylex();\}

## Simplest Example

$$
\begin{aligned}
& \text { \% \% } \\
& \text {. In ECHO; } \\
& \text { \% \% } \\
& \text { main() } \\
& \text { \{ } \\
& \text { yylex(); } \\
& \text { \} }
\end{aligned}
$$

- No definitions
- One rule
- Minimal wrapper
- Echoes input


## Strings containing aa

```
%%
(a|b)*aa(a|b)* {printf("Accept %s\n", yytext);}
[a|b]+
.|n
    ECHO;
%%
main() {yylex();}
```


## Rules

- Each has a rule has a pattern and an action
- Patterns are regular expression
- Only one action is performed
- The action corresponding to the pattern matched is performed
- If several patterns match the input, the one corresponding to the longest sequence is chosen
- Among the rules whose patterns match the same number of characters, the rule given first is preferred


## Definitions

- The definitions block allows you to name a RE
- If the name appears in curly braces in a rule, the RE will be substituted

```
DIG [0-9]
%%
{DIG}+ printf("int: %s\n", yytext);
{DIG}+"."{DIG}* printf("float: %s\n", yytext);
    /* skip anything else */
%%
main() {yylex();}
```

```
/* scanner for a toy Pascal-like language */
\% \{
\#include <math.h> /* needed for call to atof() */
\%\}
DIG [0-9]
ID [a-z][a-z0-9]*
\% \%
\(\{\mathrm{DIG}\}+\quad \operatorname{printf}(\) "Integer: \(\% \mathrm{~s}(\% \mathrm{~d}) \backslash n "\), yytext, atoi(yytext));
\(\{D I G\}+" . "\{D I G\} * \operatorname{printf}(\) "Float: \%s (\%g)\n", yytext, atof(yytext));
if|then|begin|end printf("Keyword: \%s\n",yytext);
\{ID \}
"+"|"-"|"*"|"/" printf("Operator: \%s\n",yytext);
"\{"[^\}\n]*"\}" /* skip one-line comments */
[ \(\backslash \backslash \backslash n]+\quad / *\) skip whitespace */
    printf("Unrecognized: \%s\n",yytext);
\(\% \%\)
main() \(\{\) yylex();\}
```

x character 'x' $\quad$ Flex's RE syntax
[xyz] character class, in this case, matches either an 'x', a 'y', or a 'z'
[abj-oZ] character class with a range in it; matches 'a', 'b', any letter from ' $j$ ' through 'o', or 'Z'
[^A-Z] negated character class, i.e., any character but those in the class, e.g. any character except an uppercase letter.
[^A-Z\n] any character EXCEPT an uppercase letter or a newline
$\mathbf{r}^{*} \quad z e r o$ or more r 's, where r is any regular expression
$\mathbf{r}+\quad$ one or more r's
$\mathbf{r}$ ? zero or one r's (i.e., an optional r)
\{name\} expansion of the "name" definition
"[xy]\"foo" the literal string: '[xy]"foo' (note escaped ")
l $\quad$ if $x$ is an ' a ', ' b ', ' f ', ' n ', ' r ', ' t ', or ' $v$ ', then the ANSI-C interpretation of $\backslash x$. Otherwise, a literal 'x' (e.g., escape)
rs $\quad \mathrm{RE} \mathrm{r}$ followed by RE s (e.g., concatenation)
$\mathbf{r} \mid \mathbf{s} \quad$ either an $r$ or an $s$
$\ll \mathbf{E O F} \gg$ end-of-file

