IR Models: The Vector Space Model

Lecture 7

Boolean Model Disadvantages

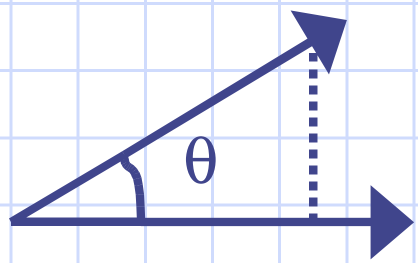
- Similarity function is boolean
  - Exact-match only, no partial matches
  - Retrieved documents not ranked
- All terms are equally important
  - Boolean operator usage has much more influence than a critical word
- Query language is expressive but complicated

The Vector Space Model

- Documents and queries are both vectors

$$\vec{d}_i = (w_{i,1}, w_{i,2} \ldots w_{i,t})$$

  - each $w_{i,j}$ is a weight for term *j* in document *i*
  - "bag-of-words representation"
- Similarity of a document vector to a query vector = cosine of the angle between them



Cosine Similarity Measure

$$sim(d_i, q) = \cos\theta$$

$$(x \cdot y = |x||y|\cos\theta)$$

$$= \frac{d_i \cdot q}{|d_i||q|} = \frac{\sum_j w_{i,j} \times w_{q,j}}{\sqrt{\sum_j w_{i,j}^2}\sqrt{\sum_j w_{q,j}^2}}$$

- Cosine is a normalized dot product
- Documents ranked by decreasing cosine value
  - sim(d,q) = 1 when d = q
  - sim(d,q) = 0 when d and q share no terms

Term Weighting

- Higher weight = greater impact on cosine
- Want to give more weight to the more "important" or useful terms
- What is an important term?
  - If we see it in a query, then its presence in a document means that the document is relevant to the query.
  - How can we model this?

Clustering Analogy

- Documents are collection of C objects
- Query is a vague description of a subset A of C
- IR problem: partition C into A and ~A
- We want to determine
  - which object features best describe members of A
  - which object features best differentiate A from ~A
- For documents,
  - frequency of a term in a document
  - frequency of a term across the collection

Term Frequency (tf) factor

- How well does a term describe its document?
  - if a term *t* appears often in a document,
    then a query containing *t* should retrieve that document
  - frequent (non-stop) words are thematic
    - flow, boundary, pressure, layer, mach

$$tf_{i,j} = \frac{f_{i,j}}{\max_j f_{i,j}}$$

$$tf_{i,j} = 1 + \log f_{i,j}$$

$$tf_{i,j} = 0.5 + \frac{0.5 \times f_{i,j}}{\max_j f_{i,j}}$$

$$tf_{i,j} = K + \frac{(1-K) \times f_{i,j}}{\max_j f_{i,j}}$$

Inverse Document Frequency (idf) factor

- A term's *scarcity* across the collection is a measure of its importance
  - Zipf's law: term frequency ≈ 1/rank
  - importance is inversely proportional to frequency of occurrence

$$idf_t = \log(1 + \frac{N}{n_t})$$

$$idf_t = \log(\frac{N - n_t}{n_t})$$

N = # documents in coll

n_t = # documents containing term t

tf-idf weighting

- A weighting scheme where

$$w_{d,t} = tf_{d,t} \times idf_t$$

  is called a *tf-idf scheme*
- tf-idf weighting is the most common term weighting approach for VSM retrieval
- There are many variations...

tf-idf Monotonicity

- "A term that appears in many documents should ***not*** be regarded as ***more important*** than one that appears in few documents."
- "A document with many occurrences of a term should ***not*** be regarded as ***less important*** than a document with few occurrences of the term."

Length Normalization

$$\frac{d_i \cdot q}{|d_i||q|}$$

- Why normalize by document length?
- Long documents have
  - **Higher term frequencies**: the same term appears more often
  - **More terms**: increases the number of matches between a document and a query
- Long documents are more likely to be retrieved
- The "cosine normalization" lessens the impact of long documents

VSM Example

d	Document vectors $\langle tf_{d,t} \rangle$										W_d
	col	day	eat	hot	lot	nin	old	pea	por	pot	
1	1.0			1.0				1.7	1.7		2.78
2								1.0	1.0	1.0	1.73
3		1.0				1.0	1.0				1.73
4	1.0			1.0						1.7	2.21
5								1.7	1.7		2.40
6			1.0		1.0						1.41
idf_t	1.39	1.95	1.95	1.39	1.95	1.95	1.95	1.1	1.1	1.39	

- q1 = *eat*
- q2 = *porridge*
- q3 = *hot porridge*
- q4 = *eat nine day old porridge*

Vector Space Model

Advantages

- Ranked retrieval
- Terms are weighted by importance
- Partial matches

Disadvantages

- Assumes terms are independent
- Weighting is intuitive, but not very formal

Implementing VSM

$$sim(q,d) = \frac{1}{W_q W_d} \sum_t w_{q,t} \times w_{d,t}, W_d = \sqrt{\sum_t w_{d,t}^2}$$

- Need within-document frequencies in the inverted list
- W_q is the same for all documents
- $w_{q,t}$ and $w_{d,t}$ can be accumulated as we process the inverted lists
- W_d can be precomputed

Cosine algorithm

1. A = {} (set of accumulators for documents)
2. For each query term t
   - Get term, f_t, and address of I_t from lexicon
   - set $idf_t = \log(1 + N/f_t)$
   - Read inverted list I_t
   - For each $<d, f_{d,t}>$ in I_t
     - If $A_d \notin A$, initialize A_d to 0 and add it to A
     - $A_d = A_d + (1 + \log(f_{d,t})) \times idf_t$
3. For each A_d in A, $A_d = A_d/W_d$
4. Fetch and return top *r* documents to user

Managing Accumulators

- How to store accumulators?
  - static array, 1 per document
  - grow as needed with a hash table
- How many accumulators?
  - can impose a fixed limit
  - quit processing I_t's after limit reached
  - continue processing, but add no new A_d's

Managing Accumulators (2)

- To make this work, we want to process the query terms in order of decreasing idf_t
- Also want to process I_t in decreasing $tf_{d,t}$ order
  - sort I_t when we read it in
  - or, store inverted lists in $f_{d,t}$-sorted order

  <5; (1,2) (2,2) (3,5) (4,1) (5,2)> *<f_t; (d, $f_{d,t}$)…>*

  <5; (3,5) (1,2) (2,2) (5,2) (4,1)> *sorted by $f_{d,t}$*

  <5; (5, 1:3) (2, 3:1,2,5) (1, 1:4)> *<f_t; ($f_{d,t}$, c:d,…)…>*

  - This can actually compress better, but makes Boolean queries harder to process

Getting the top documents

- Naïve: sort the accumulator set at end
- Or, use a heap and pull top *r* documents
  - much faster if r << N
- Or better yet, as accumulators are processed to add the length norm (W_d):
  - make first r accumulators into a min-heap
  - for each next accumulator
    - if A_d < heap-min, just drop it
    - if A_d > heap-min, drop the heap-min, and put A_d in